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# Existence results for nonlocal impulsive neutral functional integro-differential equations

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**Abstract**

In this manuscript, we establish the existence of nonlocal impulsive neutral functional integro-differential equations with bounded delay in Banach spaces. Some existence results of mild solutions to such problems are obtained under the conditions in respect of the Hausdorff's measures of noncompactness.

**Keywords:** neutral partial differential equation; nonlocal condition; Impulsive neutral integro-differential equations; Mild solutions; finite delay.

**2000 Subject Classification:** 34G10, 34K30, 34K40, 47D09.

## 1 Introduction

In recent years, the theory of impulsive differential equations provides a natural frame work for mathematical modeling of many real world phenomena, namely in control, biological and medical domains. In these models, the investigated simulating processes and phenomena are subjected to certain perturbations whose duration is negligible in comparison with the total duration of the process. Such perturbations can be reasonably well approximated as being instantaneous changes of state, or in the form of impulses. These process tend to be more suitably modeled by impulsive differential equations, which allow for discontinuities in the evolution of the state. For more details on this theory and its applications, we refer to the monographs of Bainov and Simeonov [1], Lakshmikantham et al. [13] and Samoilenko and Perestyuk [18] and the papers of [7, 8, 15, 19].

Motivated by the exertion of the after mentioned papers [7, 17], the existence of mild

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solutions to nonlocal impulsive neutral functional integro-differential equation

$$\frac{d}{dt}\Big(x(t) + g(t, x(t), x_t)\Big) = Ax(t) + \int_0^t K(t,s) f(s, x(s), x_s) ds, t \in [0, T] \tag{1.1}$$

$$x_0 = \phi + h(x), \tag{1.2}$$

$$\Delta x(t_i) = I_i(x(t_i)), \quad i = 1, 2, ..., p, 0 < t_1 < t_2 < ... < t_p < T. \tag{1.3}$$

where $A$ is the infinitesimal generator of an analytic semigroup $\{T(t) : t \geq 0\}$ of linear operators defined on a Banach space $X$, $x \in C([0, T]; X)$, and $x_t : [-q, 0] \to X$ defined by $x_t(\theta) = x(t + \theta)$ for $\theta \in [-q, 0]$; $f, g : [0, T] \times X \times C([-q, 0]; X) \to X$,$K : [0, T] \times [0, T] \to (0, +\infty)$, $h : C([0, T]; X) \to C([-q, 0]; X)$ and $0 < t_1 < t_2 < ... < t_p < T$ , $I_i : X \to X, i = 1, 2, ..., p$ are appropriate functions $T, q > 0$ are constants.

## 2 Preliminaries

Throughout this paper $X$ will represent a Banach space with norm $\|.\|$. As usual, $C([a, b]; X)$ denotes the Banach space of all continuous $X$-valued functions defined on $[a, b]$ with norm $\|x\|_{[a,b]} = \sup_{s\in[a,b]} \|x(s)\|$ for $x \in C([a, b]; X)$.

Let $A : D(A) \subset X \to X$ be the infinitesimal generator of an uniformly bounded analytic semigroup of linear operators $\{T(t) : t \geq 0\}$ on $X$ such that $0 \in \rho(A)$ and we always assume that $\|T(t)\| \leq M$ for every $t \in [0, T]$. Under these conditions it is possible to define the fractional power $(-A)^\alpha, 0 < \alpha \leq 1$, as a closed linear operator on its domain $D((-A)^\alpha)$. Furthermore, $D((-A)^\alpha)$ is dense in $X$ and the expression $\|x\|_\alpha = \|(-A)^\alpha x\|$ defines a norm on $D((-A)^\alpha)$. If $X_\alpha$ is the spaace $D((-A)^\alpha)$ endowed with the norm $\|.\|_\alpha$, then the following properties hold [16], pp.74.

**Lemma 2.1.** *Let $0 < \alpha \leq \beta \leq 1$. Then the following properties hold :*

- **(i)** *$X_\beta$ is a Banach space and $X_\beta \hookrightarrow X_\alpha$ is continuous.*
- **(ii)** *The function $s \longmapsto (-A)^\alpha T(s)$ is continuous in the uniform operator topology on $(0, \infty)$ and there exists a positive constant $C_\alpha$ such that $\|(-A)^\alpha T(t)\| \leq \frac{C_\alpha}{t^\alpha}$ for every $t > 0$.*

For more details of the semigroup theory we refer the readers to [16].

The Hausdorff's measure of noncompactness $\chi Y$ is defined by $\chi Y(B) = \inf \Big\{r > 0, B$ can be covered by finite number of balls with radii $r\Big\}$ for bounded set $B$ in any Banach space $Y$.

**Lemma 2.2.** *([2]): let $Y$ be a real Banach space and $B, C \subset Y$ be bounded, the following properties are satisfied:*

(1) *$B$ is pre-compact if and only if $\chi Y(B) = 0$;*

(2) $\chi Y(B) = \chi Y(\bar{B}) = \chi Y(convB)$ *where* $\bar{B}$ *and* $convB$ *mean the closure and convex hull of* $B$ *respectively;*

(3) $\chi Y(B) \leq \chi Y(C)$ *when* $B \subseteq C$*;*

(4) $\chi Y(B + C) \leq \chi Y(B) + \chi Y(C)$ *where* $B + C = \{x + y; x \in B, y \in C\}$*;*

(5) $\chi Y(B \cup C) \leq \max\{\chi Y(B), \chi Y(C)\}$

(6) $\chi Y(\lambda B) = |\lambda| \chi Y(B)$ *for any* $\lambda \in R$*;*

(7) *If the map* $Q : D(Q) \subseteq Y \to Z$ *is Lipschitsz continuous with constant* $k$ *then* $\chi Z(QB) \leq k\chi Y(B)$ *for any bounded subset* $B \subseteq D(Q)$*, where* $Z$ *be a Banach space;*

(8) $\chi Y(B) = \inf\{dY(B, C); C \subseteq Y \text{ be precompact}\} = \inf\{dY(B, C); C \subseteq Y \text{ be finite valued}\}$, *where* $dY(B, C)$ *means the non symmetric (or symmetric) Hausdorff distance between* $B$ *and* $C$ *in* $Y$*.*

(9) *If* $\{W_n\}_{n=1}^{+\infty}$ *is a decreasing sequence of bounded closed nonempty subsets of* $Y$ *and* $\lim\limits_{n\to+\infty} \chi Y(W_n) = 0$*, then* $\cap_{n=1}^{+\infty} W_n$ *is nonempty and compact in* $Y$*.*

The map $Q : W \subseteq Y \to Y$ is said to be a $\chi Y-$ *contraction* if there exists a positive constant $k < 1$ such that $\chi Y(Q(C)) \leq k\chi Y(C)$ for any bounded closed subset $C \subseteq W$ where $Y$ is a Banach space.

**Lemma 2.3.** *([2]): ( Darbo-Sadovskii) If* $W \subseteq Y$ *is bounded closed and convex, the continuous map* $Q : W \to W$ *is a* $\chi Y-$ *contraction, then the map* $Q$ *has at least one fixed point in* $W$*.*

In this paper we denote $\chi$ by the Hausdorff's measure of noncompactness of $X$ and denote $\chi_c$ by the Hausdorff's measure of noncompactness of $C([0;T]; X)$. To discuss the existence we need the following lemmas in this paper.

**Lemma 2.4.** *([2]) If* $W \subseteq C([0;T]; X)$ *is bounded, then* $\chi(W(t)) \leq \chi_c(W)$ *for all* $t \in [0, T]$*, where* $W(t) = \{u(t); u \in W\} \subseteq X$*. Furthermore if* $W$ *is equicontinuous on* $[0, T]$*, then* $\chi(W(t))$ *is continuous on* $[0, T]$ *and* $\chi_c(W) = sup\{\chi(W(t)), t \in [0, T]\}$.

**Lemma 2.5.** *([9, 11]): If* $\{u_n\}_{n=1}^{\infty} \subset L^1(a, b; X)$ *is uniformly integrable, then* $\chi(\{u_n(t)\}_{n=1}^{\infty})$ *is measurable and* $\chi(\left\{\int\limits_a^t u_n(s)ds\right\}_{n=1}^{\infty}) \leq \xi \int\limits_a^t \chi(\{u_n(s)\}_{n=1}^{\infty})ds$ *where* $\xi = 1$ *if* $\{u_n\}$ *is equicontinuous and* $\xi = 2$ *if* $\{u_n\}$ *is not equicontinuous.*

**Lemma 2.6.** *([2]): If* $W \subseteq C([0, T]; X)$ *is bounded and equicontinuous, then* $\chi(W(s))$ *is continuous and* $\chi(\int\limits_0^t W(s)ds) \leq \int\limits_0^t \chi(W(s))ds$ *for all* $t \in [0, T]$*,*

*where* $\int\limits_0^t W(s)ds = \left\{\int\limits_0^t x(s)ds : x \in W\right\}$.

*The* $C_0$ *semigroup* $T(t)$ *is said to be equicontinuous if* $t \to \{T(t)x : x \in B\}$ *is equicontinuous for* $t > 0$ *for all bounded set* $B$ *in* $X$*. It is known that the analytic semigroup is equicontinuous. The following lemma is obvious.*

**Lemma 2.7.** *: If the semigroup $T(t)$ is said to be equicontinuous $\eta \in L(0,T;R^+)$, then the set* $\left\{ \int_0^t T(t-s)u(s)ds, \|u(s)\| \leq \eta(s) for a.e. s \in [0,T] \right\}$ *is equcontinuous for $t \in [0,T]$.*

# 3 Existence theorem

In this section we consider the existence results of the problem (1.1)-(1.3). We define the mild solution for problem (1.1)-(1.3) by the integral equation

$$\begin{aligned} x(t) &= T(t)\Big[\phi(0) + h(x)(0) + g(0,\phi(0)+h(x)(0),\phi+h(x))\Big] - g(t,x(t),x_t) \\ &\quad - \int_0^t AT(t-s)g(s,x(s),x_s)ds + \int_0^t T(t-s)\int_0^s K(s,r)f(r,x(r),x_r)drds \\ &\quad + \sum_{0<t_i<t} T(t-t_i)I_i(x(t_i)), \qquad 0 \leq t \leq T. \end{aligned} \tag{3.1}$$

**Definition 3.1.** *A continuous function $x : [-q,T] \to X$ is said to be a mild solution to the nonlocal neutral problem (1.1)-(1.3) if $x_0 = \phi + h(x)$, for each $t \in [0,T]$ the function $s \mapsto AT(t-s)g(s,x(s),x_s)$ is integrable on $(0,t]$, and the integral equation (3.1) is satisfied.*

In this section by using the usual techniques of the Hausdorff measure of noncompactnes and its applications in differential equations in Banach spaces (see,e.g. [2], [12]) we give some existence results of the nonlocal neutral problem (1.1)-(1.3). Here we list the following hypotheses.

$(Hf)(1)$ : $f : [0,T]\times X\times C([-q,0];X) \to X$ satisfies the cartheodory-type condition, i.e., $f(.,x,\phi) : [0,T] \to X$ is measurable for all $(x,\phi) \in X \times C([-q,0];X)$ and $f(t,.) : X \times C([-q,0];X) \to X$ is continuous for a.e. $t \in [0,T]$;

(2) : There exists an integrable function $\alpha : [0,T] \to [0,+\infty)$ and a continuous nondecreasing function $\Omega : [0,+\infty) \to [0,+\infty)$ such that $\|f(t,x,\phi)\| \leq \alpha(t)\Omega(\|x\| + \|\phi\|_{[-q,0]})$ for all $(t,x,\phi) \in [0,T] \times X \times C([-q,0];X)$;

(3) : There exists an integrable function $\eta : [0,T] \to [0,+\infty)$ such that: $\chi(T(s)f(t,D_1,D_2)) \leq \eta(t)(\chi(D_1)) + \sup_{-q\leq\theta\leq 0} \chi(D_2(\theta))$ for a.e. $t,s \in [0,T]$ and any bounded subset $D_1 \subset X$ and $D_2 \subset C([-q,0];X)$, where $D_2(\theta) = \{v(\theta) : v \in D_2\}$.

$(Hf)(3)'$ : There is an integrable function $\eta : [0,T] \to [0,+\infty)$ such that $\chi(f(t,D_1,D_2)) \leq \eta(t)(\chi(D_1)) + \sup_{-q\leq\theta\leq 0} \chi(D_2(\theta))$ for a.e. $t \in [0,T]$ and any bounded subset $D_1 \subset X$ and $D_2 \subset C([-q,0];X)$, then we can get the obvious result:

$(Hg)$ : There exists $0 < \beta < 1$ such that $g$ is $X_\beta$-valued, $(-A)^\beta g(.)$ is continuous and there exist positive constants $c_1, c_2$ and $L_g$ such that $\left\|(-A)^\beta g(t,x,\phi)\right\| \leq c_1(\|x\| + \|\phi\|_{[-q,0]}) + c_2$

and $\left\|(-A)^{\beta}g(t,x_1,\phi_1)-(-A)^{\beta}g(t,x_2,\phi_2)\right\| \le L_g(\|x_1-x_2\|+\|\phi_1-\phi_2\|_{[-q,0]})$ for all $t\in[0,T], x, x_1, x_2 \in X$ and $\phi,\phi_1,\phi_2 \in C([-q,0];X)$.

$(Hh)(1):$ $h : C([0,T];X) \to C([-q,0];X)$ is Lipschitz continuous in the following sense: there exists a positive constants $L_h$ such that $\|h(x)-h(y)\|_{[-q,0]} \le L_h \|x-y\|_{[0,T]}$ for all $x,y\in C([0,T];X)$;

$(Hh)(2):$ $h$ is uniformly bounded, i.e., there is a positive constant $N$ such that $\|h(x)\|_{[-q,0]} \le N$ for all $x,y\in C([0,T];X)$;

$(Hi)(1):$ There exists a constant $d_i$ such that $\|I_i(x)\| \le d_i, i=1,s,...,p.$
$d=\max\{d_i\}, i=1,2,...,p.$

$(Hi)(2):$ $I_i : X\to X$ is continuous and there exists constant $l_i$ such that
$\|I_i(x)-I_i(y)\| \le l_i\|x-y\|, i=1,2,...,p., for\ all\ \ x,y\in X.$

$(Hk)(1):$ For each $t\in[0,T], K(t,.)$ is measurable on $[0,t]$, and
$K(t)=ess\ \sup\{|K(t,s)| : 0\le s\le t\}$ is bounded on $[0,T]$.

$(2):$ The map $t\mapsto K_t$ is continuous from $[0,T]$ to $L^{\infty}(0,T;R^{+})$, here $K_t(s)=K(t,s)$

$(Hc)':$ $2c_1(\left\|(-A)^{-\beta}\right\|+\frac{C_{1-\beta}T^{\beta}}{\beta})+TKM\int_0^T\alpha(s)ds\liminf_{k\to\infty}\frac{\Omega(2k)}{k}<1,$
where $K=\sup_{0\le t\le T}K(t)$.

Now we are in the position to state our main result of this section.

**Theorem 3.1.** *Assume the hypotheses* $(Hf)(1),(2),(3)',(Hg),(Hh),(Hi),(Hk)$ *and* $(Hc)'$ *are satisfied. Then for every* $\phi\in C([-q,0];X)$*, the problem (1.1)-(1.3) has atleast one mild solution provided* $L_0+8TMK\int_0^T\eta(t)dt<1,$

**Proof.** Considee the map $\Gamma : C([-q,T];X)\to C([-q,T];X)$ defined by $\Gamma=\Gamma_1+\Gamma_2$, where

$$\Gamma_1x(t)=\begin{cases}\phi(t)+h(x)(t), & t\in[-q,0]\\ T(t)\Big[\phi(0)+h(x)(0)+g(0,\phi(0)+h(x)(0),\phi+h(x))\Big]-g(t,x(t),x_t) & \\ -\int_0^t AT(t-s)g(s,x(s),x_s)ds+\sum_{0<t_i<t}T(t-t_i)I_i(x(t_i)), & t\in[0,T],\end{cases}$$

$$\Gamma_2x(t)=\begin{cases}0, & t\in[-q,0],\\ \int_0^t T(t-s)\int_0^s K(s,r)f(r,x(r),x_r)drds, & t\in[0,T].\end{cases}$$

As in the proof of Theorem 3.1 [7, 17], we can verify (with some obvious modifications) that $\Gamma$ is a continuous $\chi_c$-contraction, and that there is a $k\in N$ such that $\Gamma$ maps $B_k$ into itself. Thus Darbo-Sadovskii's fixed point theorem can be used to get a fixed point of $\Gamma$, which is a mild solution of (1.1)-(1.3).

Here we only need to prove that there is a $k \in N$ such that $\Gamma(B_k) \subset B_k$ and to estimate $\chi_c \Gamma_2(W)$ for every bounded subset $W \subset C([-q,T];X)$.

Suppose for each $k \in N$ there is $x^k \in B_k$ and $t^k \in [-q,T]$ such that $\left\|\Gamma x^k(t^k)\right\| > k$, then if $t^k \in [-q,0]$ we have

$$k \leq \left\|\Gamma x^k(t^k)\right\| \leq \|\phi\|_{[-q,0]} + N \tag{3.2}$$

and if $t^k \in [0,T]$ we have

$$k < \left\|\Gamma x^k(t^k)\right\| \leq M\Big[\|\phi(0)\| + N + \left\|(-A)^{-\beta}\right\| (2c_1(\|\phi\|_{[-q,0]} + N) + c_2) + \sum_{0<t_i<t} d_i\Big]$$
$$+ (\left\|(-A)^{-\beta}\right\| + \frac{C_{1-\beta}T^\beta}{\beta})(2c_1 k + C_2) + TMK\int_0^t \alpha(s)ds.\Omega(2k).$$

Denote by $L_k$ the right hand side of the above inequality, then we have
$k < \left\|\Gamma x^k(t^k)\right\| \leq \max(\|\phi\|_{[-q,0]} + N, L_k)$ Divided by $k$ on both sides of (3.2) and then take lim inf as $k \to \infty$ we have $2c_1(\left\|(-A)^{-\beta}\right\| + \frac{C_{1-\beta}T^\beta}{\beta}) + TKM\int_0^T \alpha(s)ds \liminf_{k\to\infty} \frac{\Omega(2k)}{k} \geq 1$, which contradicts the hypatheses $(Hc)'$. Hence there is a $k \in N$ such that $\Gamma(B_k) \subset B_k$.

Now, for every bounded subset $W \subset C([-q,T];X)$ and any $\epsilon > 0$, we can take a sequence $\{x_n\}_{n=0}^\infty \subset W$ such that $\chi_c(W) \leq 2\chi_c(\{x_n\}_{n=0}^\infty) + \epsilon$ (see, e.g., [4] pp.125). From $(Hf)(2)$ and $(Hk)$ we know that $\{K(s,.)f(.,x_n(.),x_n.)\}_{n=1}^\infty$ is uniformly integrable on $[0,s]$ for $s \in (0,T]$. By using Lemma 2.2, Lemma 2.4-2.7, $(Hf)(3)'$ and $(Hk)$,
we have $\chi_c(\Gamma_2 W) \leq 8TMK\chi_c(W)\int_0^T \eta(s)ds$.
Since $L_0 + 8TMK\int_0^T \eta(s)ds < 1$ and
$\chi_c(\Gamma W) = \chi_c(\Gamma_1 W) + \chi_c(\Gamma_2 W) \leq (L_0 + 8TMK\int_0^T \eta(s)ds)\chi_c(W)$, we obtain that $\Gamma$ is a $\chi_c$-contraction. Using Lemma 2.2, we get a fixed point $x$ of $\Gamma$, which is a mild solution of (1.1)-(1.3). The proof is complete.

From the proof Theorem 3.1, we can see that the condition $(HK)(1)$ can be replaced by

$(Hk)$ : $(1)'$ For each $t \in (0,T]$, $K(t,.)$ is measurable on $[0,t]$, and $K(t) = ess\sup\{|K(t,s)| : 0 \leq s \leq t\}$ is integrable on $[0,T]$,

which is slightly weaker than $(HK)(1)$. Denote by $K_1 = \int_0^T K(t)dt$, then we get the following obvious result:

**Theorem 3.2.** *Assume the hypotheses $(Hf)(1),(2),(3)',(Hg),(Hh),(Hi)$ and $(Hk)(1)',(2)$ are satisfied. Then for every $\phi \in C([-q,0];X)$, the problem (1.1)-(1.3) has atleast one mild* *solution if* $L_0 + 8MK_1 \int_0^T \eta(s)ds < 1$, *and*

$2c_1(\left\|(-A)^{-\beta}\right\| + \frac{C_{1-\beta}T^{\beta}}{\beta}) + K_1 M \int_0^T \alpha(s)ds \lim\inf_{k\to\infty} \frac{\Omega(2k)}{k} < 1.$

We can also remove the restriction that the map $h$ is uniformly bounded.

**Theorem 3.3.** *Assume the hypotheses* $(Hf)(1),(2),(3)',(Hg),(Hh),(Hi),(Hk)$ *and* $(Hh)(1)$ *are satisfied. Then for every* $\phi \in C([-q,0];X)$*, the nonlocal neutral problem (1.1)-(1.3) has atleast one mild solution if* $L_0 + 8TMK \int_0^T \eta(t)dt < 1$, *and*

$2c_1(\left\|(-A)^{-\beta}\right\| + \frac{C_{1-\beta}T^{\beta}}{\beta}) + M \lim\inf_{k\to\infty} (\int_0^T \alpha(s)ds \frac{\Omega(2k)}{k} + (1 + c_1 \left\|(-A)^{-\beta}\right\|)\frac{\gamma(k)}{k}) < 1.$

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